Indian J. Phys. **48**, 537–542 (1974)

Ionospheric absorption calculation by Jaeger's formula with finite limits

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(*Received* 17 *August* 1973)

The limits of integration in Jaeger's formula has been changed from infinity to finite values for calculating transmission absorption at different layers of the ionosphere. The integral has been evaluated for different altitudes with a wave of frequency 5MHz for solar zenith angle 28°. The values of absorption calculated with Jaeger's formula using the above values of the functions are compared graphically with those calculated by other formulae.

1. INTRODUCTION

The absorption suffered by any radiowave for transmission through a region in the ionosphere may be given by Jaeger's (1947) approximate formula

$$Kds = \frac{4.133\nu_0 H}{c \sec \chi} \; \frac{f_c^2}{f^2} \qquad \ldots \quad (1)$$

Such absorption values could also be calculated from the relation by Ratcliffe (1962) using Rawer's expression for μ,

$$\int Kds = \int \frac{\nu}{2c} \cdot \frac{f_c^2 \sec^{\frac{1}{2}} \chi}{f(f^2 - f_c^2 \sec^{\frac{1}{2}} \chi)^{\frac{1}{2}}}, \qquad \ldots \quad (2)$$

It was shown by Misra et al (1973a, 1973b) that the values of absorption calculated from eqs. (1) and (2) using 5 MHz as well as 30 MHz wave at solar zenith angle χ (= 28°) are different. The values from eq. (1) are much higher than those from eq. (2) which is evident from figure 1. It may be pointed out here that both eqs. (1) and (2) are derived from Lorentz's theory using the same relation viz.,

$$\int Kdh = \int \frac{\nu}{2c} \; \frac{1-\mu^2}{\mu} \, dh. \qquad \ldots \quad (3)$$

Furthermore, the same values of ν, c, χ etc. are used for calculating absorption in both the relations. Also the same parameters are in use for calculating f_c for a particular layer. Still the two relations give different values of absorption calculated by them. The possible causes may be the following

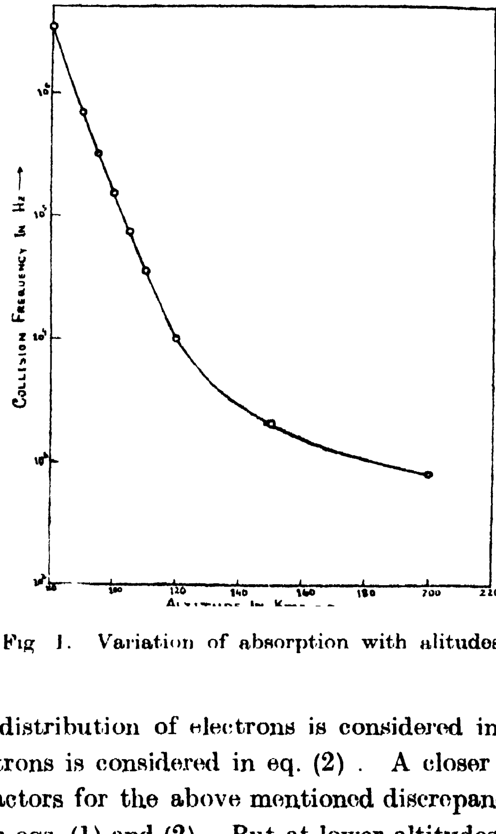

Fig 1. Variation of absorption with alitudes.

(a) Parabolic distribution of electrons is considered in eq. (1) while linear distribution of electrons is considered in eq. (2) . A closer scrutiny shows that this is one of the factors for the above mentioned discrepancy among the corresponding values from eqs. (1) and (2). But at lower altitudes this is not the most important cause

(b) In eq. (1) the wave travels from a layer where $\mu \approx 1$ upto a place from where the wave will be reflected back. In other words the author of eq. (1) is interested in studying absorption effect from $-\infty$ to $+\infty$ above and below the datum level from which the wave is projected. But the authors of the present paper are interested in calculating absorption effect from layer to layer in the ionosphere. The wave thus travels from one finite altitude to another finite altitude. Hence the limits of integration in evaluating $F\ (f_c/f)$ should be changed to finite values. This is the most important cause of the discrepancy in the values of absorption shown in figure 1. It is important to mention here that the values of $F(f_c/f)$ should be evaluated for particular layer with finite limits of integration.

2. Calculation and Results

From Lorentz's theory the absorption suffered by a wave may be given by eq. (3). Substituting Chapmanian distribution of electron density i.e.,

$$N = N_0 \exp \tfrac{1}{2}(1-Z-\sec \chi e^{-z}), \tag{4}$$

in eq. (3) and also using the relations

$$Z = \frac{h-h_0}{H}, \tag{5}$$

and

$$\nu = \nu_0 e^{-z} \tag{6}$$

we get,

$$\int K dh = \frac{\nu_0 f_0^2 H}{2cf^2} \int \frac{\exp \frac{1}{2}(1-3Z-\sec \chi e^{-z})}{\left(1-\left(\frac{f_0^2}{f^2}\right)\exp \frac{1}{2}(1-Z-\sec \chi e^{-z})\right)^{\frac{1}{2}}} \tag{7}$$

For convenience the notations $b = (f^{02}/f^2)\sqrt{2e}$ and $y = (\frac{1}{2}\sec \chi)^{\frac{1}{2}} e^{-\frac{1}{2}z}$ may be substituted in eq. (7) which gives

$$\int_{h_0}^{h} K dh = \frac{\nu_0 H}{c \sec \chi} \int_{y_1}^{y_2} \frac{2by^2 e^{-y^2} dy}{(1-bye^{-y^2})^{\frac{1}{2}}}$$

$$= -\frac{\nu_0 H}{c \sec \chi} \psi\left(\frac{f_c}{f}\right). \tag{8}$$

$$\text{where } \psi\left(\frac{f_c}{f}\right) = -\int_{y_1}^{y_2} \frac{2by^2 e^{-y^2}}{(1-bye^{-y^2})^{\frac{1}{2}}} dy\,. \tag{9}$$

Since the integral is cumbersome in the present form and the values of y and b are such that bye^{-y^2} is less than unity the expression under radical sign may be expanded binomially. The higher terms may be neglected particularly at lower altitudes. Thus

$$\psi\left(\frac{f_c}{f}\right) = 2b \int_{y_1}^{y_2} y^2 e^{-y^2} \Big[1+\tfrac{1}{2}bye^{-y^2}+\frac{1.3}{2.4}\,b^2y^2e^{-2y^2} + \frac{1.3.5}{2.4.6}\,b^3y^3e^{-3y^3}+\ldots\Big]dy$$

$$= \int_{y_1}^{y^2} [2by^2e^{-y^2}+b^2y^3e^{-2y^2}+\frac{3}{4}b^3y^4e^{-3y^2}+\frac{5}{8}b^4y^5e^{-4y^2}+\ldots]\, dy.$$

Let the integrations be performed term wise. Let the first, second, third, fourth etc. terms be represented respectively by ψ_i, ψ_{ii}, ψ_{iii}, ψ_{iv} etc. then

$$\psi_i = 2b \int_{y_1}^{y_2} y^2 e^{-y^2}\, dy.$$

This type of integrals could not be evaluated directly. Hence the integral is evaluated by graphical means

$$\psi_{ii} = b^2 \int_{y_1}^{y_2} y^2 e^{-2y^2} dy.$$

Integrating by parts it becomes,

$$\psi_{ii} = \frac{b^2}{4}\left[y^2 e^{-2y^2} + \tfrac{1}{2} e^{-2y^2} \right]_{y_1}^{y_2}$$

Now

$$\psi_{iii} = \frac{3}{4} b^3 \int_{y_1}^{y_2} y^4 e^{-3y^2}\, \mathrm{d}y.$$

Proceeding as in ψ_i we also evaluate ψ_{iii} graphically. Similarly,

$$\psi_{iv} = \frac{5}{8} b^4 \int_{y_1}^{y_2} y^5 e^{-4y_2}\, \mathrm{d}y.$$

Integrating by parts,

$$\psi_{iv} = \frac{5}{64} b^4 \left[y^4 e^{-4y^2} + \frac{1}{2} y^2 e^{-4y^2} + \frac{1}{8} e^{-4y_2} \right]$$

Hence the total value of the function will be

$$\psi\left(\frac{f_c}{f}\right) = \psi_i + \psi_{ii} + \psi_{iii} + \psi_{iv} + \cdot \qquad \ldots \quad (10)$$

ψ_i, ψ_{ii}, ψ_{iii}, ψ_{iv} etc. may be exactly evaluated for definite values of y_1 and y_2 at a particular region with a wave of particular frequency for a certain solar zenith angle and be summed up to give the total value of $\psi(f_c/f)$. As the higher terms are negligible for lower altitudes these terms are not evaluated. But these terms should be included at greater altitudes. From actual calculations it is seen that ψ_{ii} is considerable above 80 kms., ψ_{iii} above 100 kms., and ψ_{iv} above 110 kms. and so on. The values of the function for different altitudes will be communicated in tabular form.

The function $\psi(f_c/f)$ for different layers are evaluated for 5 MHz waves at solar zenith angle $\chi(=28°)$ taking the values of ν from the graph, shown in figure 2, drawn with the data from Chapman & Little (1957). The scale height H is also taken from the curve, shown in figure 3, drawn by using Rocket Panel data (1952). Thus using the above values of ν, H etc. absorption of waves travelling through different layers of the ionosphere are calculated from

$$\int_{h_0}^{h} K dh = \frac{\nu_0 H}{c \sec \chi} \psi(f_c/f). \qquad \ldots \quad (11)$$

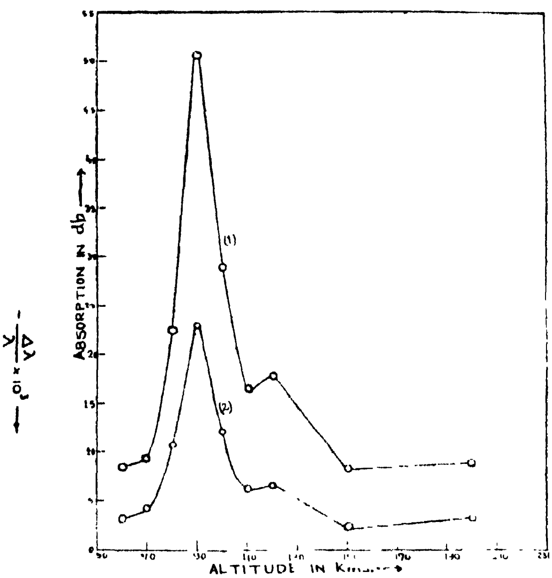

Fig. 2. Variation of collision frequency with altitudes.

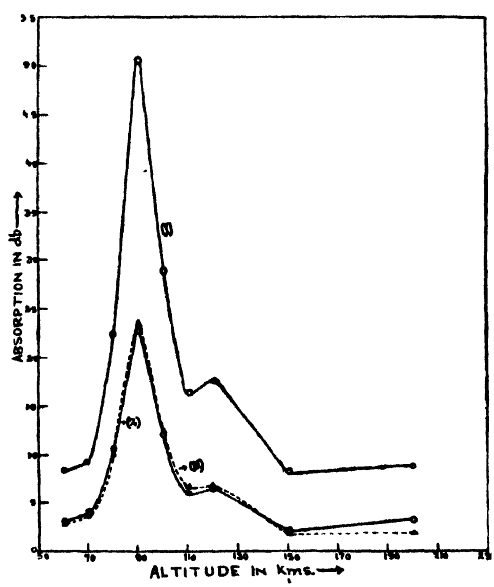

Fig. 3. Variation of scale height with altitudes.

The results are plotted in a graph and compared with the values from eqs. (1) and (2). It may be pointed out here that the absorption values calculated from eq. (11) are lower than those calculated from eq. (1) and are more close to those from eq. (2) which could be seen from figure 4. The discrepancy which still remains

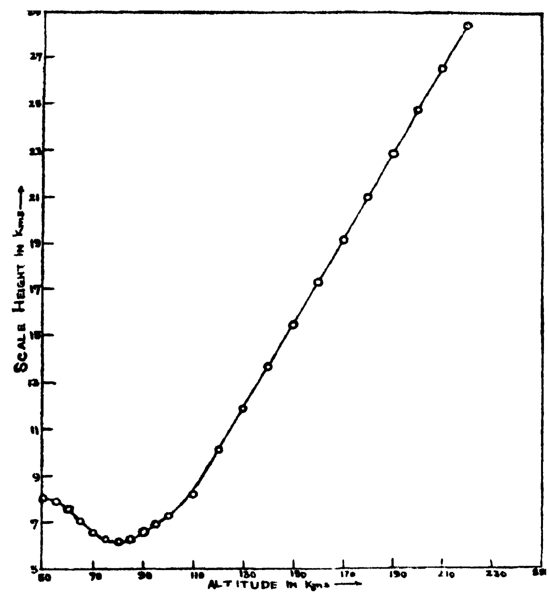

Fig. 4. Plot of absorption in db against altitudes in Kms.

may be thought of as due to the values of N, the electron density, taken in eqs (11) and (2) and also due to the approximations made in the evaluation of the integrations etc. The discrepancy still remaining is not so serious as in figure 1. Thus newly calculated values of $\psi(f_c/f)$ may be used for calculation of absorpton from layer to layer.

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